

Maths in Science

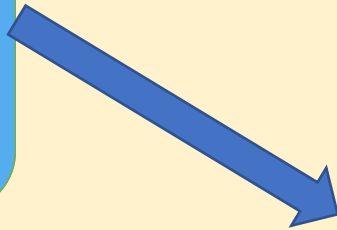
- 1. What is the difference between procedural and declarative knowledge?**
- 2. How should the nature of knowledge affect the decisions you make about its teaching?**
- 3. Case study: How I use cognitive science findings to teach mathematics in science?**



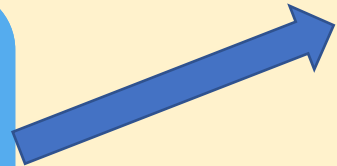
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Knowledge and Practice

The nature of
knowledge varies



Pupils need lots of
practice to master
new knowledge.



Type of practice depends
on the type of knowledge



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Procedural vs Declarative

Declarative: *know that*

- Substances are made out of atoms
- Atoms have mass
- W stands for Weight

Procedural: *know how*

- Steps for performing a calculation
- Drawing an ion from an atom (add brackets and charge)



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Decisions for Teaching

Knowing that '*performing calculations*' is **procedural knowledge** can help us make decisions about its teaching:

1. Teach necessary declarative knowledge explicitly
2. Break down procedural knowledge into constituent steps
3. Worked examples
4. Scaffolding: faded examples & misconceptions
5. Interleaved practice



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Principle 1: Declarative Knowledge

- What does the calculation represent?
- What facts are needed to perform and understand calculation?
- Issue - complex web:
 - Wait for mastery, or move to procedural?



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Substances are made
out of atoms



Atoms have

mass

Amounts

No. of molecules

M_r

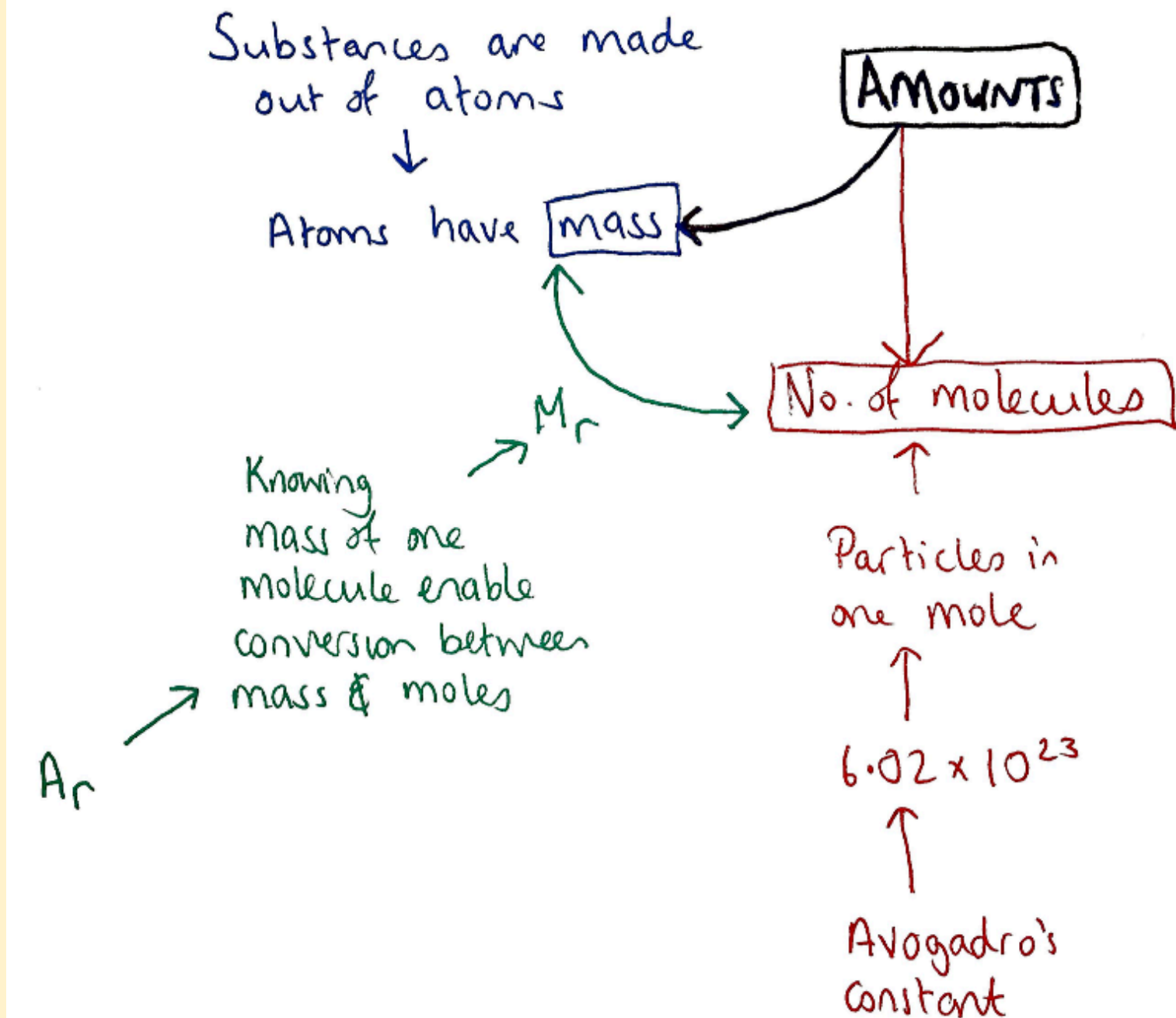
Knowing
mass of one
molecule enable
conversion between
mass & moles

A_r

Particles in
one mole

$$6.02 \times 10^{23}$$

Avogadro's
constant



Principle 2: Procedural Steps

- Simplest possible steps
 - Pupils don't need names or descriptions if complex
1. Box the value to be calculated & underline given values
 2. List all values, with units
 3. Write equation which unites values
 4. Check units – convert if necessary
 5. Substitute
 6. Combine values if possible, then rearrange
 7. Calculate final answer & write units.

$$\begin{aligned} n &= ? \text{ mol} \\ m &= 88 \text{ g} \\ M_r &= 12 + (2 \times 16) \\ &= 12 + 32 \\ &= 44 \end{aligned}$$

$$\begin{aligned} n &= \frac{m}{M_r} \\ n &= \frac{88}{44} \\ n &= \underline{\underline{2 \text{ mol}}} \end{aligned}$$



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Principle 2: Procedural Steps

- Practice components separately at first
 - Drill: state the standard unit for each variable
 - Drill: convert these into standard units
- Then combine

Teacher: What is the symbol for current?

Pupil: I

Teacher: Capital or lower case?

Pupil: Upper case.

Teacher: What are the units for current?

Pupil: Amps

Teacher: What letter can we use for the units of current?

Pupil: Capital A.



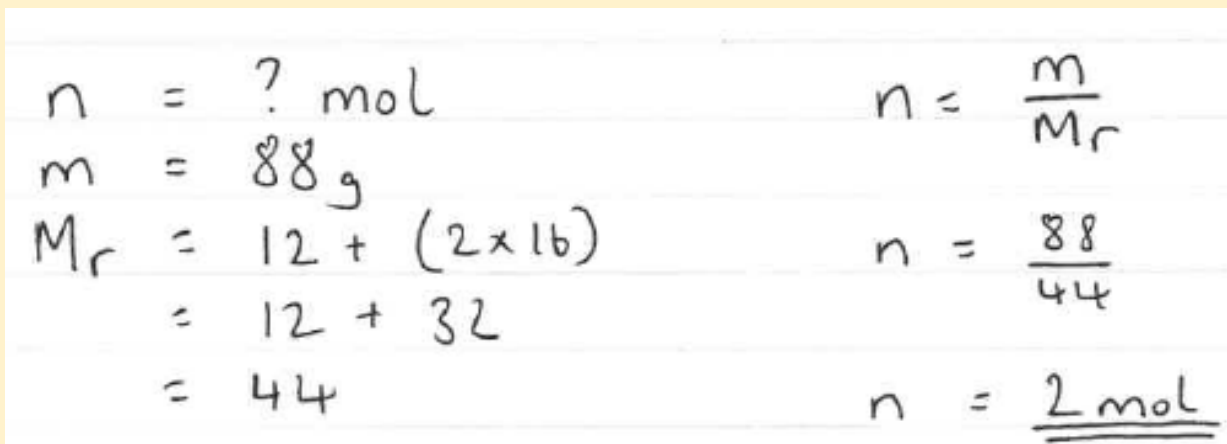
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Principle 3: Worked Examples

- Visualiser
- 1st time: no questions; 2nd time: questions
 - Standardise the format
 - Hand-written examples in textbook

Calculating moles examples [D]

Carbon dioxide (CO₂) and carbon monoxide (CO) can both be made by reacting carbon with oxygen. Relative atomic masses (A_r): C = 12, O = 16. Calculate the number of moles of 88 g of carbon dioxide (CO₂).



Handwritten calculations for finding the number of moles of 88g of carbon dioxide (CO₂):

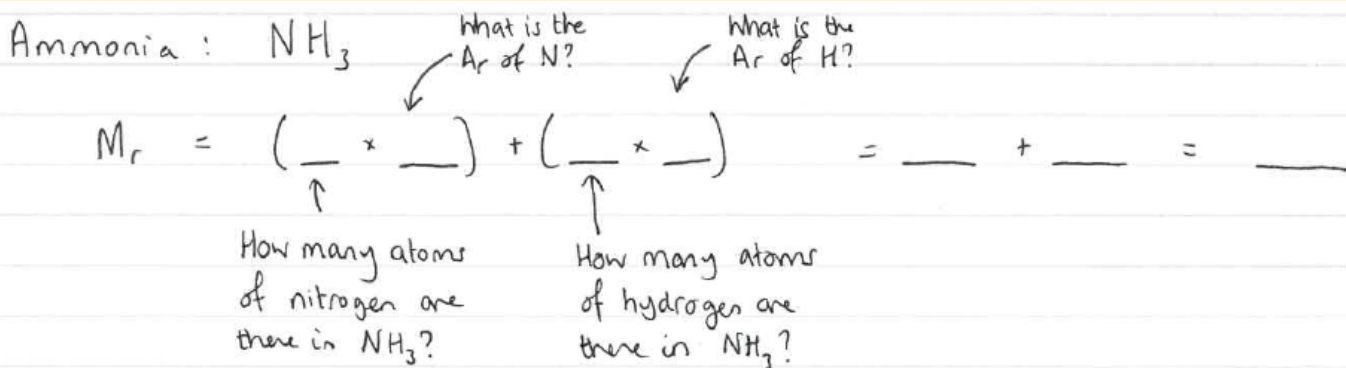
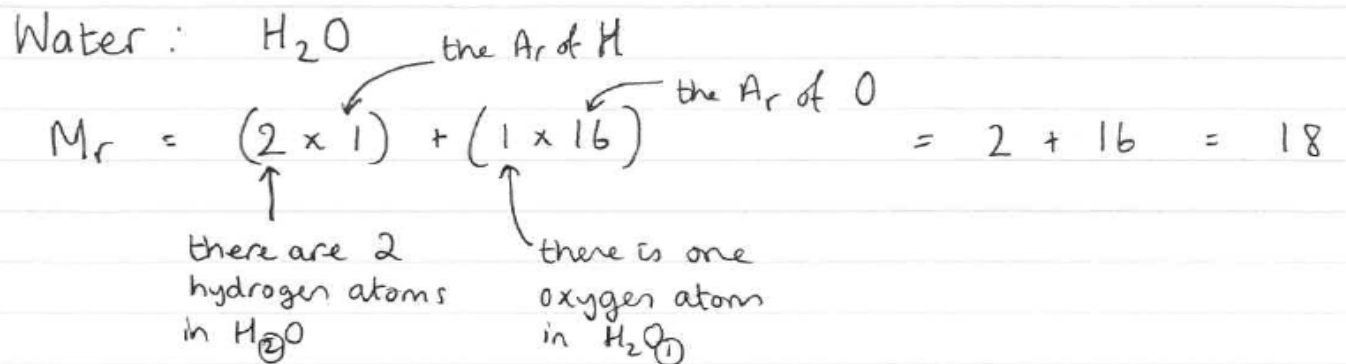
$$\begin{aligned} n &= ? \text{ mol} \\ m &= 88 \text{ g} \\ M_r &= 12 + (2 \times 16) \\ &= 12 + 32 \\ &= 44 \end{aligned}$$
$$\begin{aligned} n &= \frac{m}{M_r} \\ n &= \frac{88}{44} \\ n &= \underline{\underline{2 \text{ mol}}} \end{aligned}$$



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Principle 4: Scaffolded Practice

- Faded examples enable the practice of one component at a time
- Labelled examples → faded
- Misconception examples



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Calcium carbonate : CaCO_3

$$M_r = _ + _ + (_ \times _)$$

$$= _ + _ + _$$

$$= \underline{\underline{_}}$$

Nitrogen : N_2

$$M_r = (\underline{2} \times _)$$

Top tip: always put the number of atoms first

$$= \underline{\underline{_}}$$

Nitric acid: HNO_3

$$M_r = _ + _ + (\underline{3} \times _)$$

$$= _ + _ + _$$

$$= \underline{\underline{_}}$$

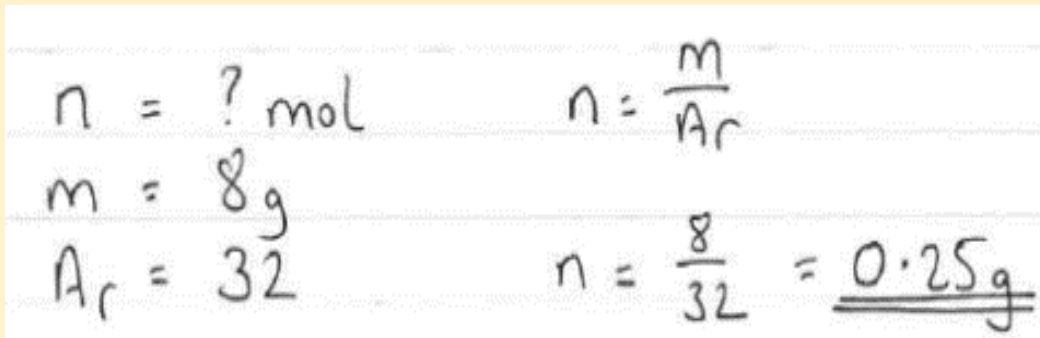


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Principle 4: Scaffolded Practice

- Faded examples enable the practice of one component at a time
- Labelled examples → faded
- Misconception examples

**Calculate the number of moles in 8 g of sulphur.
Spot the mistake!**


$$\begin{array}{ll} n = ? \text{ mol} & n = \frac{m}{A_r} \\ m = 8 \text{ g} & \\ A_r = 32 & n = \frac{8}{32} = \underline{\underline{0.25 \text{ g}}} \end{array}$$



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BUG

Calculate the mass of sodium nitrate (NaNO_3) that will be made using 17g of sodium oxide.

a



b

17g

?g

c

1

:

2

(Na₂O)

$$n = ? \text{ mol}$$

$$n = \frac{m}{M_r}$$

$$m = 17\text{g}$$

$$M_r = (2 \times 23) + 16$$

$$= 46 + 16$$

$$= 62$$

$$n = \frac{17}{62} = 0.27419...$$

$$n = 0.2742 \text{ mol}$$

d

e

1

:

2

$$0.2742 \text{ mol}$$

$$0.548 \text{ mol}$$

(NaNO₃)

$$n = 0.548 \text{ mol}$$

$$n = \frac{m}{M_r}$$

$$m = ? \text{ g}$$

$$M_r = 23 + 14 + (3 \times 16)$$

$$= 23 + 14 + 48$$

$$= 85$$

$$0.548 = \frac{m}{85}$$

$$m = 0.548 \times 85$$

$$= 46.58 \text{ g}$$

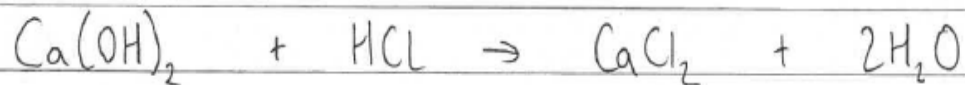
$$\approx 46.6 \text{ g}$$

f

BUG

Calculate the mass of calcium chloride produced when 7.4 grams of calcium hydroxide (Ca(OH)_2) reacts with excess hydrochloric acid.

a



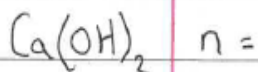
b

 g

 g

c

 :



n =

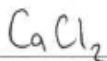
d

m =

 $M_r =$

e

 :



n =

f

m =

 $M_r =$

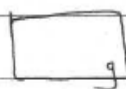
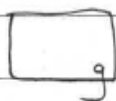
BUG

Calculate the mass carbon monoxide (CO) that can be made from 5 grams of carbon.

a



b



c



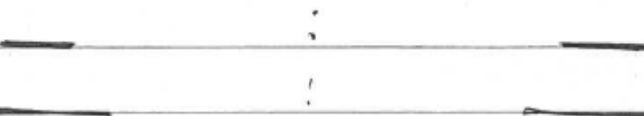
d

$n =$

$m =$

$M_r =$

e



f

$n =$

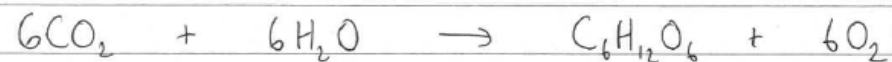
$m =$

$M_r =$

BUG

Calculate the mass of glucose a plant can make if it has 24 grams of carbon dioxide and an excess of water.

a



b

c

d

$$n =$$

$$m =$$

$$M_r =$$

e

f

$$n =$$

$$m =$$

$$M_r =$$

Principle 5: Interleave / Mixed Practice

- > 2 formula? ... Mix it up!
- Practice crucial step: selection of correct formula

Equation number	Word equation	Symbol equation
1	weight = mass × gravitational field strength (g)	$W = m g$
2	work done = force × distance (along the line of action of the force)	$W = F s$
3	force applied to a spring = spring constant × extension	$F = k e$
4	distance travelled = speed × time	$s = v t$
5	acceleration = $\frac{\text{change in velocity}}{\text{time taken}}$	$a = \frac{\Delta v}{t}$
6	resultant force = mass × acceleration	$F = m a$
7 HT	momentum = mass × velocity	$p = m v$
8	kinetic energy = $0.5 \times \text{mass} \times (\text{speed})^2$	$E_k = \frac{1}{2} m v^2$
9	gravitational potential energy = mass × gravitational field strength (g) × height	$E_p = m g h$
10	power = $\frac{\text{energy transferred}}{\text{time}}$	$P = \frac{E}{t}$
11	power = $\frac{\text{work done}}{\text{time}}$	$P = \frac{W}{t}$
12	efficiency = $\frac{\text{useful output energy transfer}}{\text{total input energy transfer}}$	
13	efficiency = $\frac{\text{useful power output}}{\text{total power input}}$	
14	wave speed = frequency × wavelength	$v = f \lambda$
15	charge flow = current × time	$Q = I t$
16	potential difference = current × resistance	$V = I R$
17	power = potential difference × current	$P = V I$
18	power = $(\text{current})^2 \times \text{resistance}$	$P = I^2 R$
19	energy transferred = power × time	$E = P t$
20	energy transferred = charge flow × potential difference	$E = Q V$
21	density = $\frac{\text{mass}}{\text{volume}}$	$\rho = \frac{m}{V}$



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Final thoughts

- Procedural before mastery of declarative:



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Final thoughts

For moles, the declarative knowledge is:

1. All substances - matter - are made out of atoms.
2. All atoms have mass.
3. To describe the amount of a substance - such as water - we can either:
 1. Count the number of water molecules
 2. Measure the mass of the substance
 3. (Measure the volume of a substance)
4. Mass and number of molecules are both valid ways of describing amounts, but are useful or appropriate for different reasons.
5. Measuring the mass of a substance can be used to calculate the number of molecules of that substance, provided that we know the mass of one molecule.
6. The total mass divided the mass of one molecule tells us the number of molecules.
7. The known mass of each particle is known as the relative atomic mass (A_r) for single atoms, and relative formula mass (M_r) for molecules (or ratios). The periodic table supplies us with this information.
8. Avogadro's constant is used to define one mole: 6.02×10^{23} molecules.
9. Avogadro's constant helps us to write numbers of molecules more concisely, since everyday amounts of substances exceed this number of particles.
10. So, number of moles is another way of saying number of molecules.
11. Therefore: mass (m); number of moles (n); and relative formula mass (M_r) are related to each other: $n = m / M_r$
12. In this way, the two ways of describing amounts of a substance: number of molecules and mass, are interchangeable.

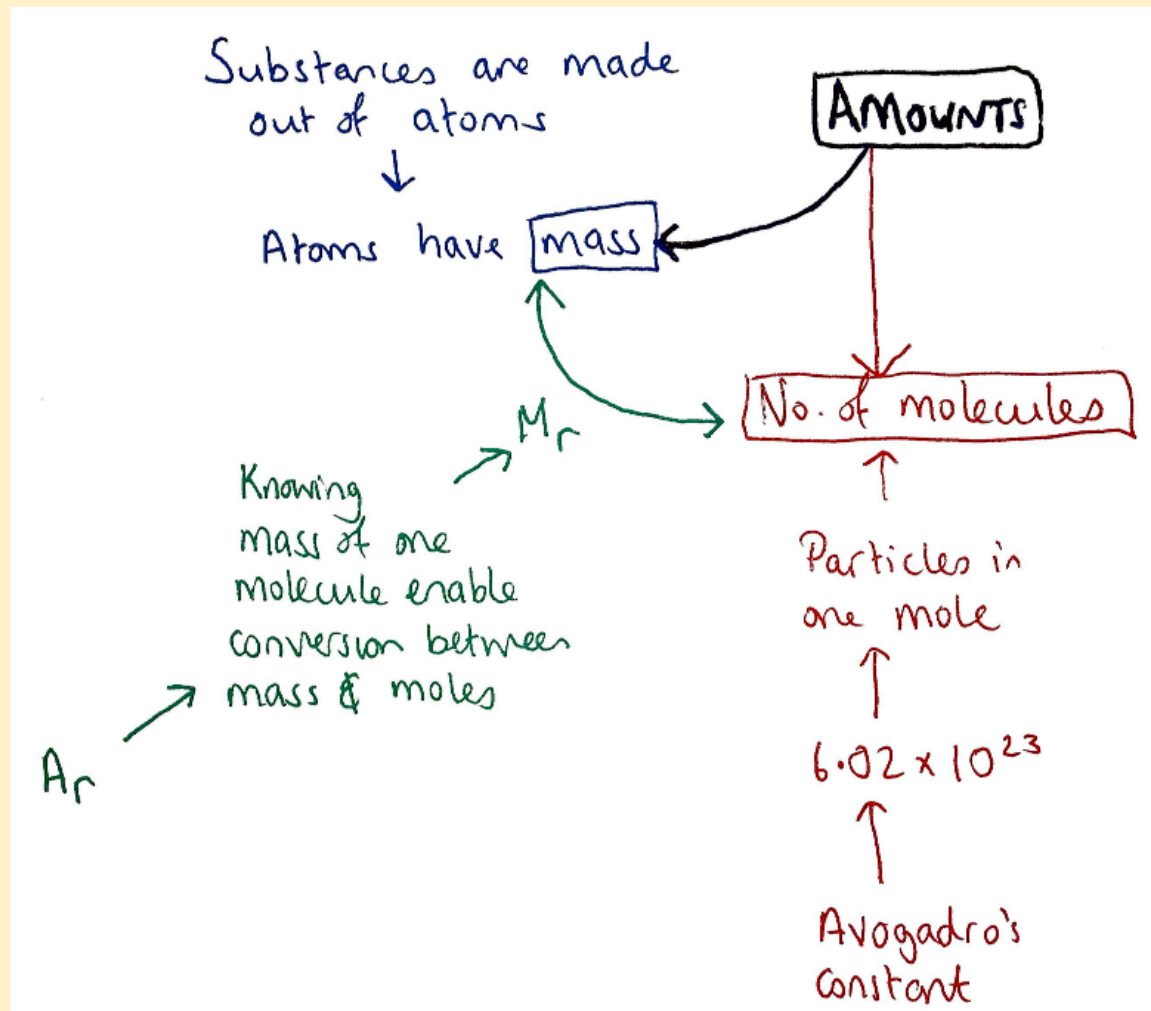


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Final thoughts

- Procedural before mastery of declarative... **dual-coding?**



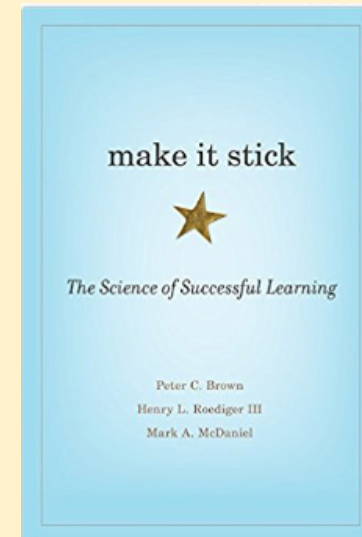
$$n = \frac{m}{M_r}$$



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References

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- Brown, P.C., Roediger III, H.L. and McDaniel, M.A., (2014) *Make it stick*. Harvard University Press.



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Questions?



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